

**A DOCKAGE DISCOUNT SCHEDULE FOR WHITE WHEAT**

by

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## SUMMARY

Last year, some of the Portland wheat exporters started to discount dockage in the grain they receive. The discounts started at .8% dockage and increased 1¢ for each additional .5% dockage above .8%. This discount schedule penalizes the delivery of high dockage wheat and should help to reduce dockage. However, it has some serious deficiencies. It encourages the blending of wheat with less than .8% dockage with higher dockage wheat at inland terminals. It guarantees that most wheat reaching Portland will have dockage as close to .8% as possible. The current discount schedule makes it impossible to isolate "clean" wheat at inland terminals. Even if the inland terminals cleaned their wheat, it would still be profitable to blend in "dirty" wheat to bring the average close to .8% when they ship to Portland. The current schedule also fails to reward farmers who have cleaner than average wheat and makes how the farmer's crop happens to be split into lots an important determinant of the total discount.

Several discount schedules that correct these deficiencies are proposed. It is argued that the discounts should start at zero dockage, should increase with each .1% increase in dockage, and that the increments in the discount schedule should decline slightly as dockage increases. This latter result is necessary to discourage blending.

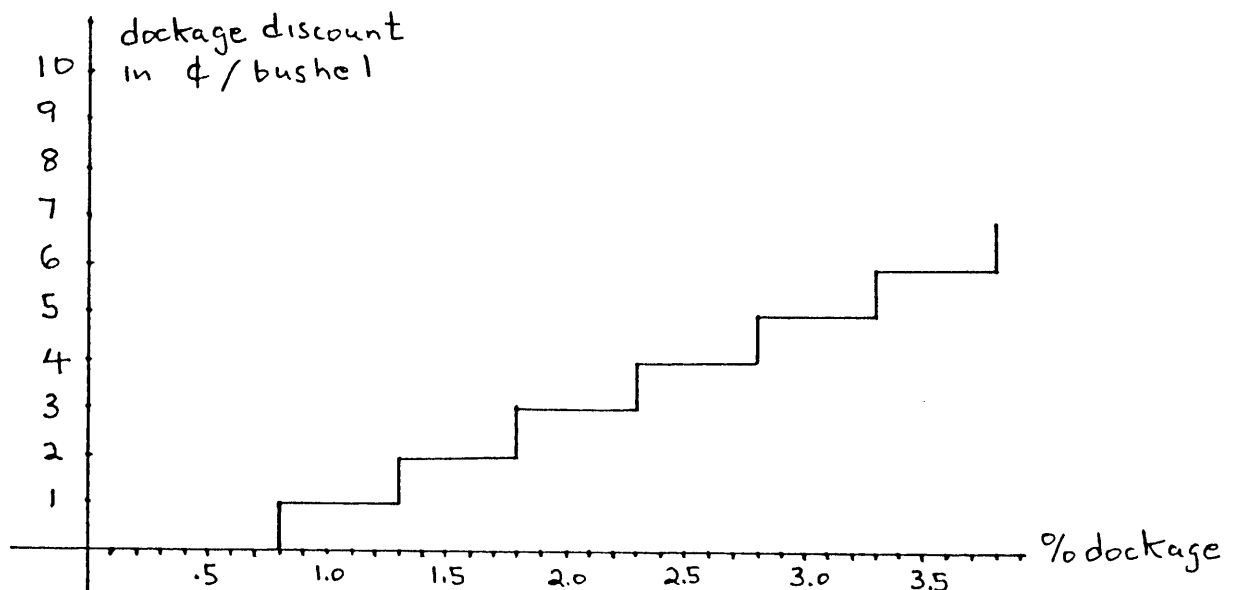
## A DOCKAGE DISCOUNT SCHEDULE FOR WHITE WHEAT

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Until last year, exporters did not discount dockage when they purchased white wheat from farmers. They did deduct the weight of the dockage in figuring the net bushels that they paid for. The only cost of dockage to farmers was the cost of paying the transportation to Portland and the handling and storage costs on the dockage. Since these costs are usually small, farmers had little incentive to minimize dockage.

Last year, several exporters announced that they would start discounting dockage in the white wheat they buy. The discount is \$.01 per bushel for each .5% of dockage above .8%. For example, wheat that has .7% dockage is not discounted, wheat with 1.2% dockage is discounted \$.01 per bushel, and wheat with 2% dockage is discounted \$.03 per bushel.



Since some customers are asking for cleaner wheat, the wheat industry needs a dockage discount schedule. The new schedule does penalize dockage and gives an incentive for farmers to do what they can to reduce dockage. However, the schedule fails to meet some reasonable criteria that a good dockage discount schedule should satisfy. For example, it encourages the blending of high and low dockage lots at the point of first delivery so that very few shipments to exporters have dockage much less than .8%. It fails to reward farmers and inland terminals that have "clean" wheat and, consequently, fails to provide a supply of "clean" wheat for customers that want it. Finally, if implemented at the farm level, it is possible for a farmer who delivers less total dockage to pay a bigger dockage penalty than a farmer who delivers more dockage. For example, a farmer whose whole crop averages .4% dockage but has one lot with 2% dockage will pay a bigger dockage discount than a farmer whose crop is in one lot that averages .75% dockage. It seems reasonable that the dockage penalty should depend on how much total dockage is delivered and not on how a farmer's crop happens to be divided up into lots.

In the next section, four goals that a dockage discount schedule should meet will be discussed and some explicit criteria for a good dockage discount schedule will be developed. In the final section a dockage discount schedule that better meets these criteria will be proposed.

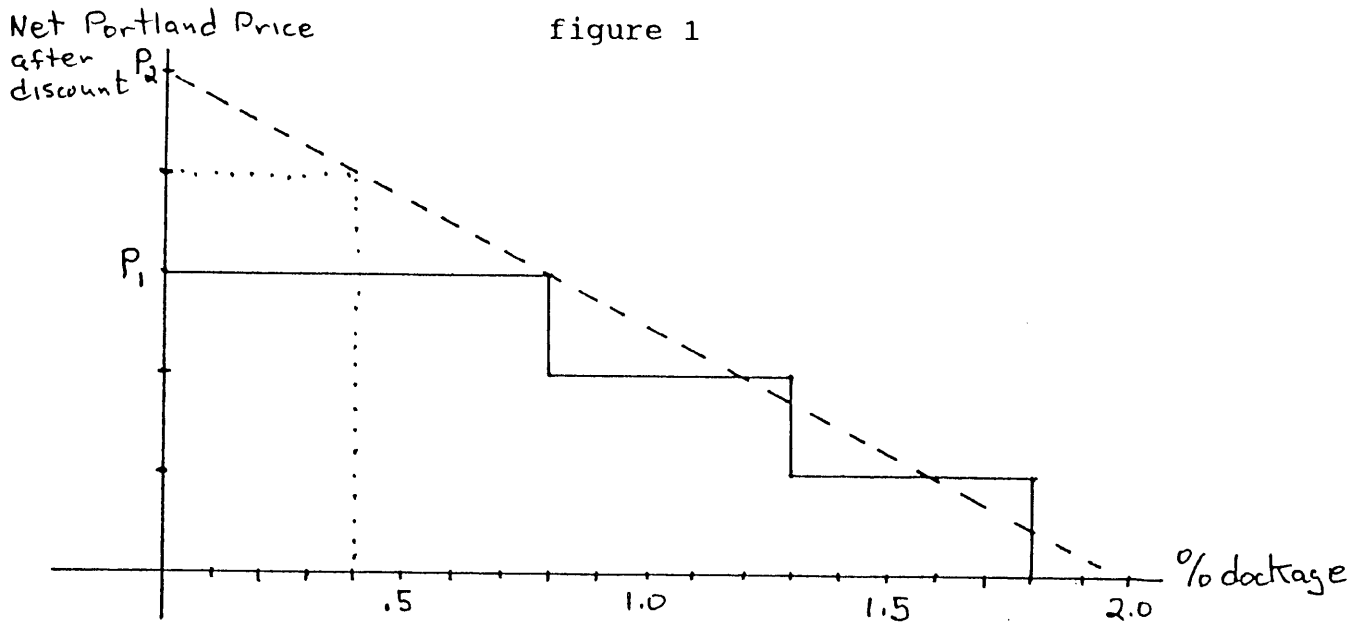
### What Should a Good Dockage Discount Schedule Do?

Some objectives of a good dockage discount schedule include:

1. The Discount Schedule Should Penalize Farmers Who Deliver High Dockage and Provide An Incentive for Them to Reduce Dockage in Their Wheat - Any discount schedule that increases as dockage increases should provide an incentive for farmers to reduce dockage. However, since the current discount schedule starts at .8%, it provides no incentive to reduce dockage to farmers with less than .8% dockage in their wheat.
2. The Discount Schedule Should Reward Farmers With Cleaner Than Average Wheat - Farmers often resist a new discount schedule and advocate the use of premiums as incentives to induce them to provide the desired qualities. However, if properly constructed, a discount schedule can provide an increase in the income of farmers with above average quality. When most white wheat is sold, the buyer expects dockage of .8% or more. The quoted Portland price of wheat is for wheat with approximately .8% dockage. If buyers really value cleaner wheat, the Portland price for 0% dockage wheat should be higher than .8% dockage wheat. Hence, if wheat was sold on the basis of zero dockage with a discount schedule for dockage above zero, the base Portland price should be higher. Even with a dockage discount schedule, the farmer with below average dockage may receive a higher net price if the increase in the base price exceeds the

discount he pays.

For example, assume a dockage discount schedule that discounts the price by .25¢ for each .1% dockage starting at zero dockage. Assume that exporters sold wheat on the basis of zero dockage with this discount schedule for deliveries of dockage above zero. If customers value cleaner wheat, zero dockage wheat should be worth more than the .7% average dockage that they are receiving now. If, for example, the Portland price of zero dockage wheat was 2¢ higher, a farmer producing wheat with .4% dockage would receive 1¢ more per bushel for his wheat even though he is paying a dockage discount of  $4 \times .25¢ = 1¢$  per bushel. The farmer's return in this case is the same as it would be if the base Portland price was for .8% dockage wheat and a premium of .25¢ was paid for each .1% that dockage was below .8%.



A pure discount schedule can provide a premium to farmers with cleaner than average wheat if it starts at zero (or near zero)

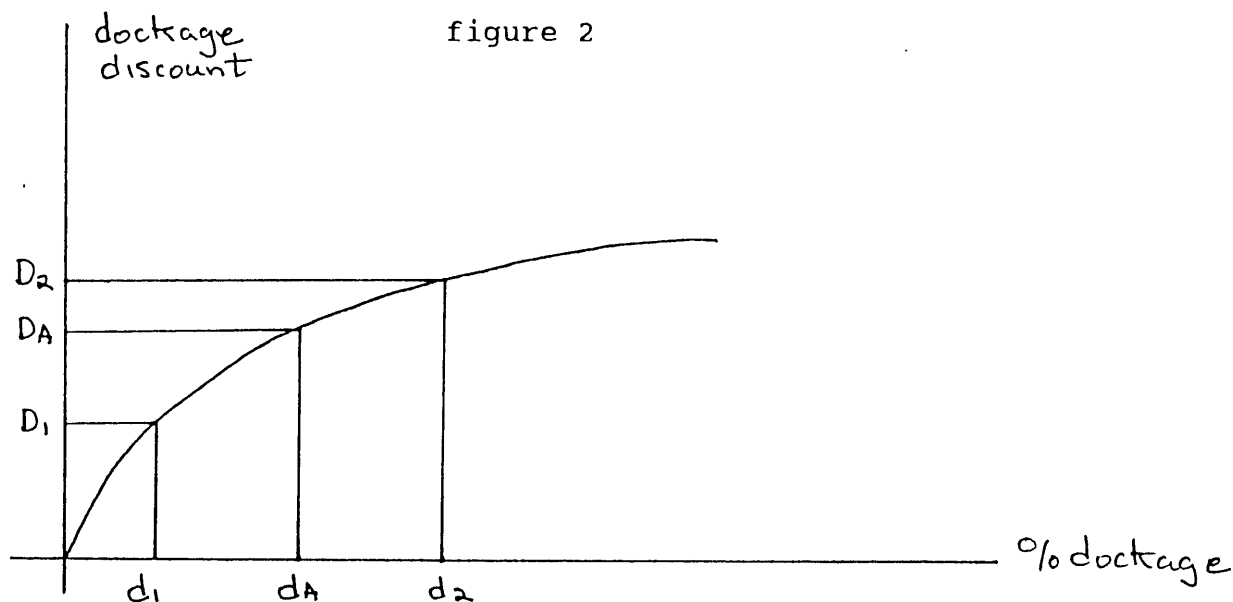
and causes the base Portland price to increase enough. Since the current dockage discount schedule starts at .8% and white wheat continues to be sold using .8% dockage as a basis, the current discount schedule provides no benefit for farmers with cleaner than average wheat.

3. The Discount Schedule Should Discourage the Blending of High and Low Dockage Wheat by Inland Grain Handlers - Most farmers initially sell their wheat to cooperatives or other inland grain terminals who then resell the wheat to exporters. Under the new dockage discount schedule, it is profitable for farmers and these inland terminals to blend high dockage lots with low dockage lots to get average dockage under .8%. No good warehouseman would ship a barge with dockage much below .8% unless he had no wheat with higher dockage. The announced schedule guarantees that little wheat will reach Portland with much less than .8% dockage.

Suppose an inland terminal invested in cleaning machinery and cleaned all the dockage out of some of its wheat. With the current dockage schedule, it would be profitable to blend this cleaned wheat with any wheat available that had dockage above .8%. Since buyers of white wheat differ on their dockage specifications, it ~~is~~ seems desirable for the dockage discount schedule to provide incentives to keep "clean" and "dirty" wheat separate at the inland terminals. A supply of "clean" wheat should be developed for those customers that want it. The blending that occurs should take place at the export terminals,

not at the point of first delivery.

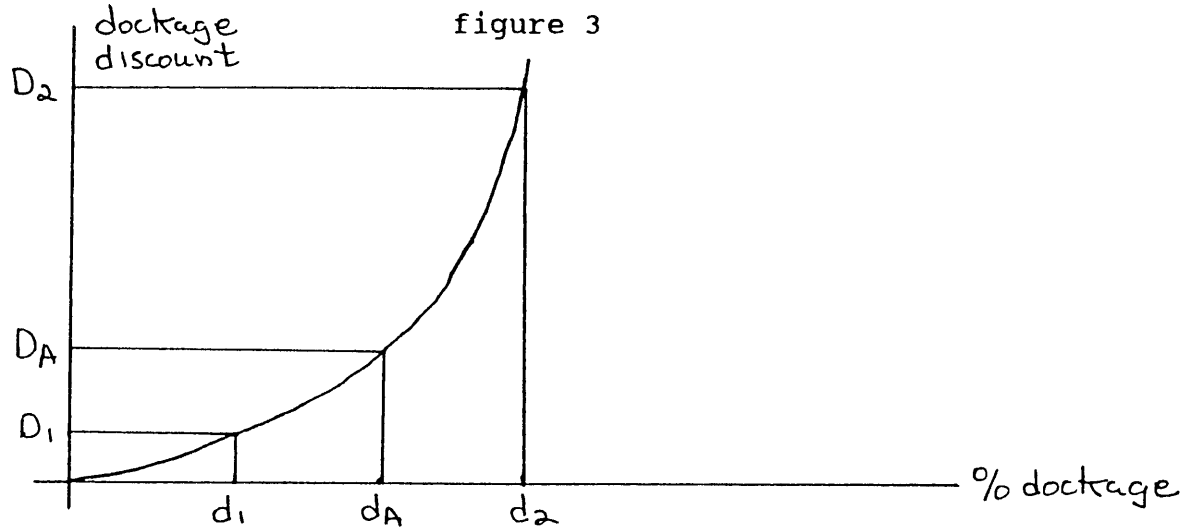
In order to discourage blending, the dockage discount per bushel should increase as the percentage of dockage increases but the rate of increase should decline as dockage increases. To understand this, consider blending two lots of equal size. Assume that the lower dockage lot has a dockage percentage of  $d_1$  and the higher dockage lot has a dockage percentage of  $d_2$ . If the lots are blended the dockage percentage of the blended lot,  $d_A$ , will be the average of  $d_1$  and  $d_2$ .



As can be seen from figure 2, if the rate of increase in the discount schedule declines as dockage increases, the reduction in the discount of the high dockage lot,  $D_2 - D_A$ , is smaller than the increase in the discount of the low dockage lot,  $D_A - D_1$ . Hence, it is profitable to keep the lots separated.

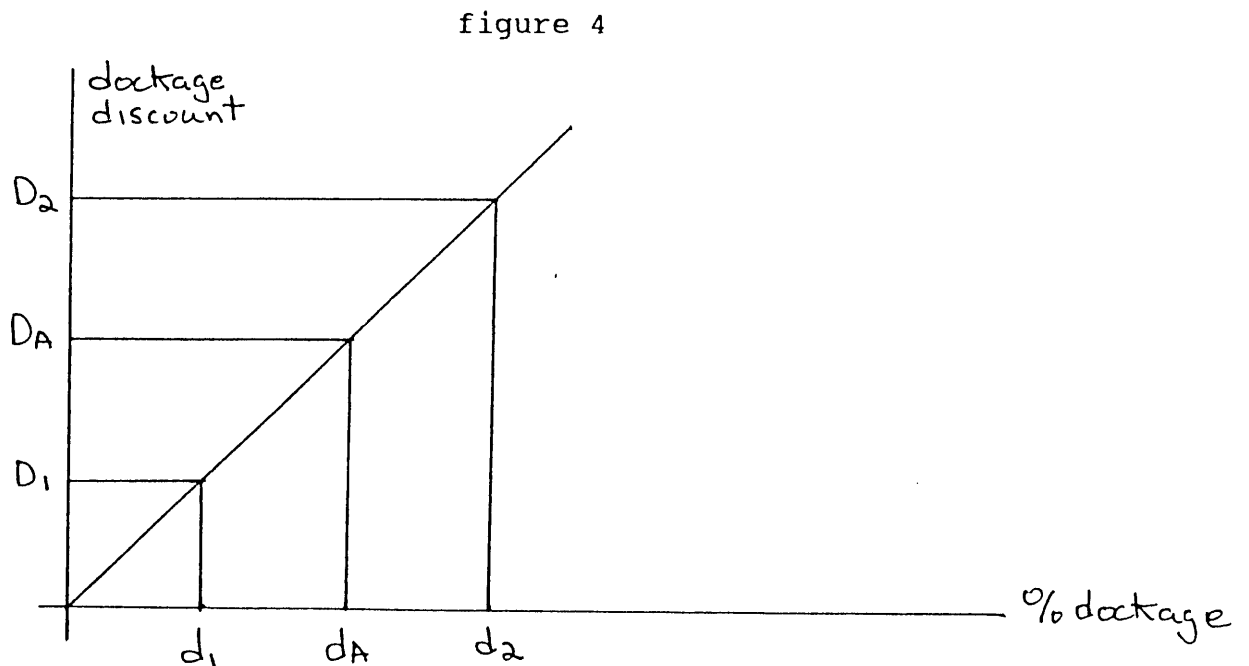
If the rate of increase of the discount schedule increases, it will be profitable to blend lots of different dockage.





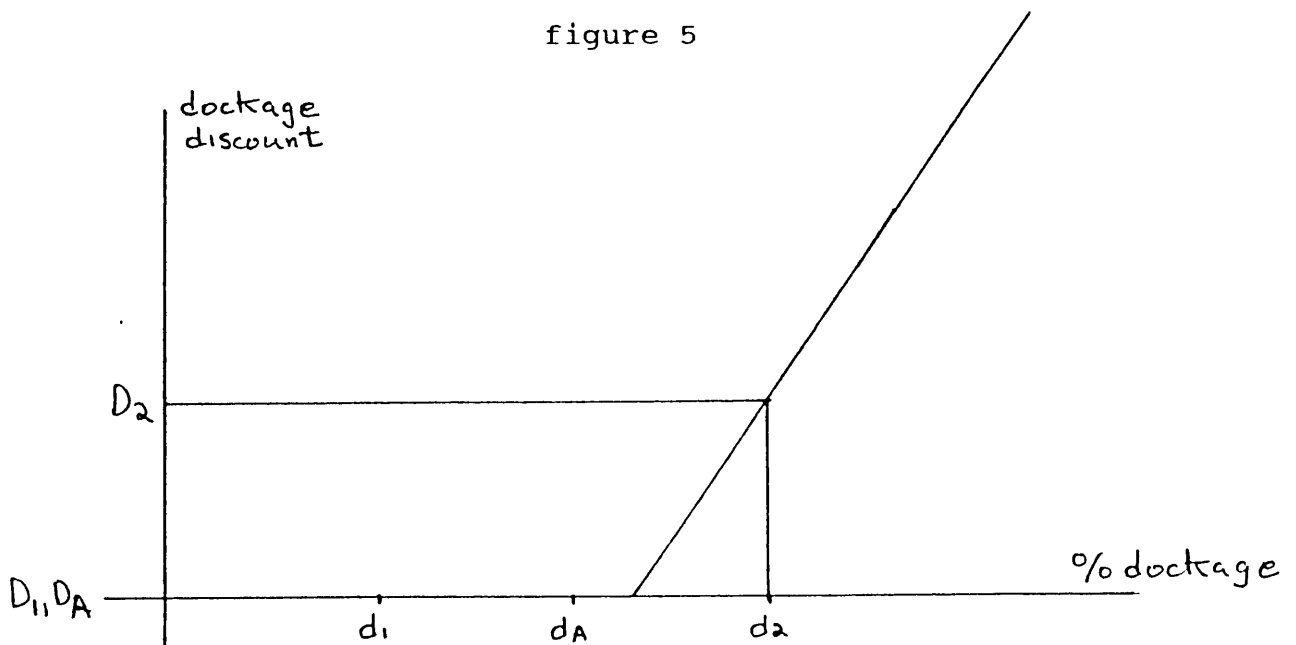
In this case  $D_2 - D_A$  is greater than  $D_A - D_1$ , so the reduction in dockage discount on the high dockage lot exceeds the increase in dockage on the low dockage lot and blending of the lots is profitable.

If the discount schedule is a straight line which starts at zero, grain handlers will be indifferent between blending lots or keeping them segregated.

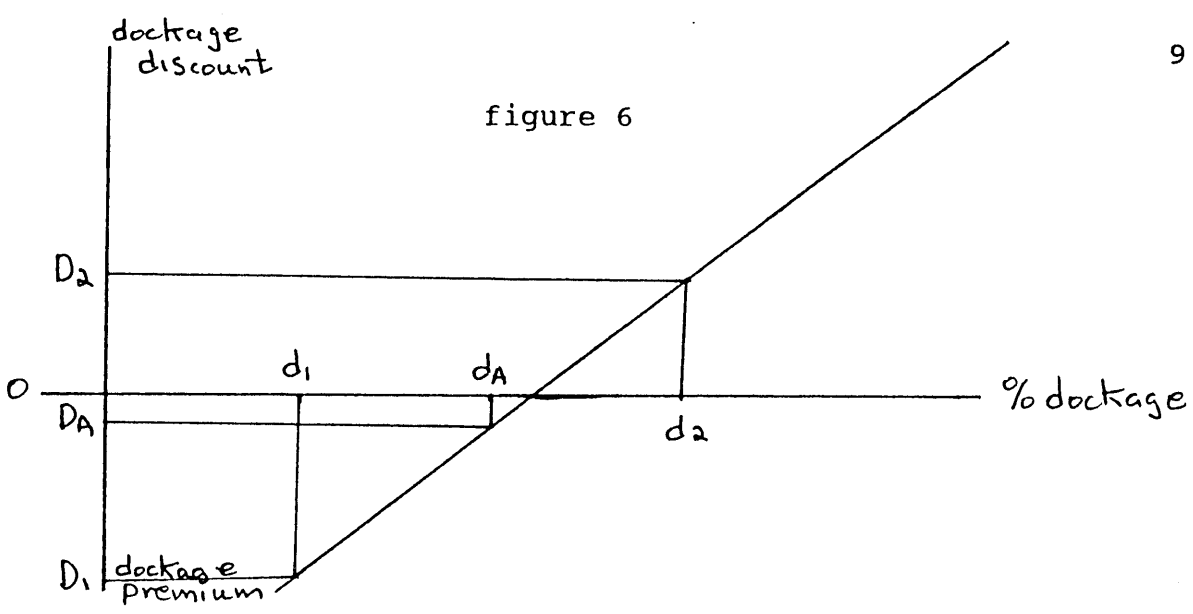


If, like the current schedule, the discounts start at some point above zero, it will be profitable to blend wheat in the zero discount range with higher dockage wheat. In this case the rate of increase of the discount schedule suddenly increases when the discounts start. As in the case considered earlier where the increments in the discount schedule increased as dockage increased, blending will be profitable.

figure 5

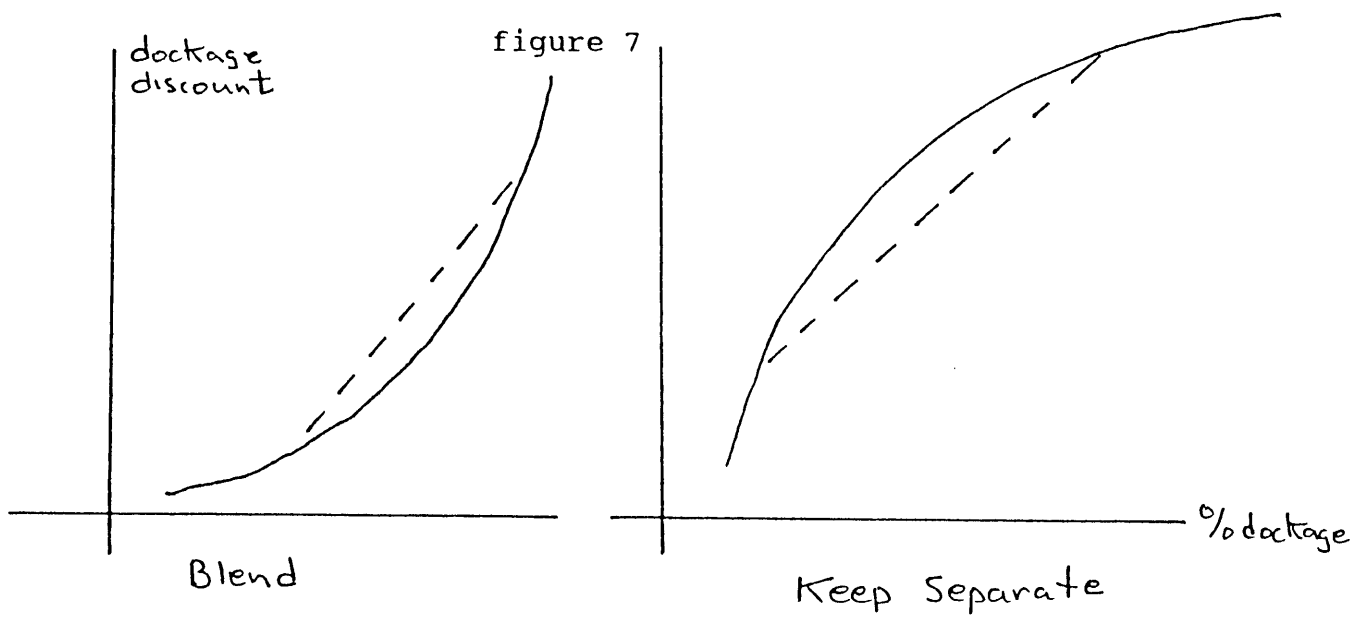


The reduction in dockage discounts of the high dockage lot clearly exceeds the increase in dockage of the low dockage lot since the latter is zero. This incentive to blend can be reduced or eliminated if a premium is added when the discount schedule hits zero. If premiums are offered, the discount schedule does not have to start at zero to prevent blending.



In this case the reduction in premium of the low dockage lot,  $D_1 - D_A$ , equals the premium on the high dockage lot after blending plus the discount before blending,  $D_2 - D_A$ , so there is no incentive to blend.

An easy way to check whether a discount schedule encourages blending lots or promotes keeping them separate is to connect two points on the schedule. If the straight line between the point lies above the discount schedule, it is profitable to blend the lots. If the line is below the discount schedule, it is profitable to keep them segregated.



In summary, in order for a discount schedule to encourage keeping lots separate, it should start at zero and its rate of increase as dockage increases should decline. If it starts at zero and the rate of increase is constant, i.e., its graph is a straight line, grain handlers will be indifferent between blending and segregating lots.

4. Farmers Who Deliver More Total Dockage Should Be Penalized More Than Farmers Who Deliver Less Total Dockage - Since the current dockage schedule gives no "credit" for wheat with dockage less than .8%, a farmer with low average dockage for his total crop but with one small high dockage lot can pay a bigger dockage penalty than another farmer who delivers more total dockage.<sup>1</sup> The current schedule has the undesirable feature that, when implemented at the farm level, it causes the size distribution of a farmer's lots to be important in determining the total discount. How the crop happens to be split into lots should not affect the dockage penalty. Total dockage should determine the farmer's total discounts.

This problem will be eliminated if the weighed average of the per bushel dockage discounts of the different lots equals the

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<sup>1</sup> For example, consider two farmers each with 1000 bushels of wheat. The first farmer's wheat is in two lots. One lot has 900 bushels in it and dockage of .2%. The other lot has 100 bushels in it and dockage of 2%. The second farmer has all his wheat in one lot with dockage of .7%. Under the current discount schedule, the first farmer will pay a penalty of  $100 \times 3\text{¢} = \$3$  and deliver  $3.8 \times 60\text{lbs} = 228$  lbs of dockage. The second farmer will deliver  $7 \times 60\text{lb} = 420$  lbs of dockage and pay no dockage penalty.

discount per bushel of the average dockage of the total crop. This condition will be met if the graph of the discount schedule is a straight line starting at zero.

#### CONCLUSION

Four reasonable criteria for a dockage discount schedule for white wheat have been developed.

1. The discounts should increase as the percentage of dockage increases to provide an incentive to reduce dockage.
2. The discounts should start at zero dockage to reward farmers whose wheat has below average dockage.
3. The discounts should increase at a decreasing rate as the percentage of dockage increases to provide an incentive to keep high and low dockage lots separate.
4. The discounts should increase at a constant rate so dockage penalties depend on total dockage delivered and not on how wheat happens to be split into lots.

The third and fourth criteria obviously conflict with each other.

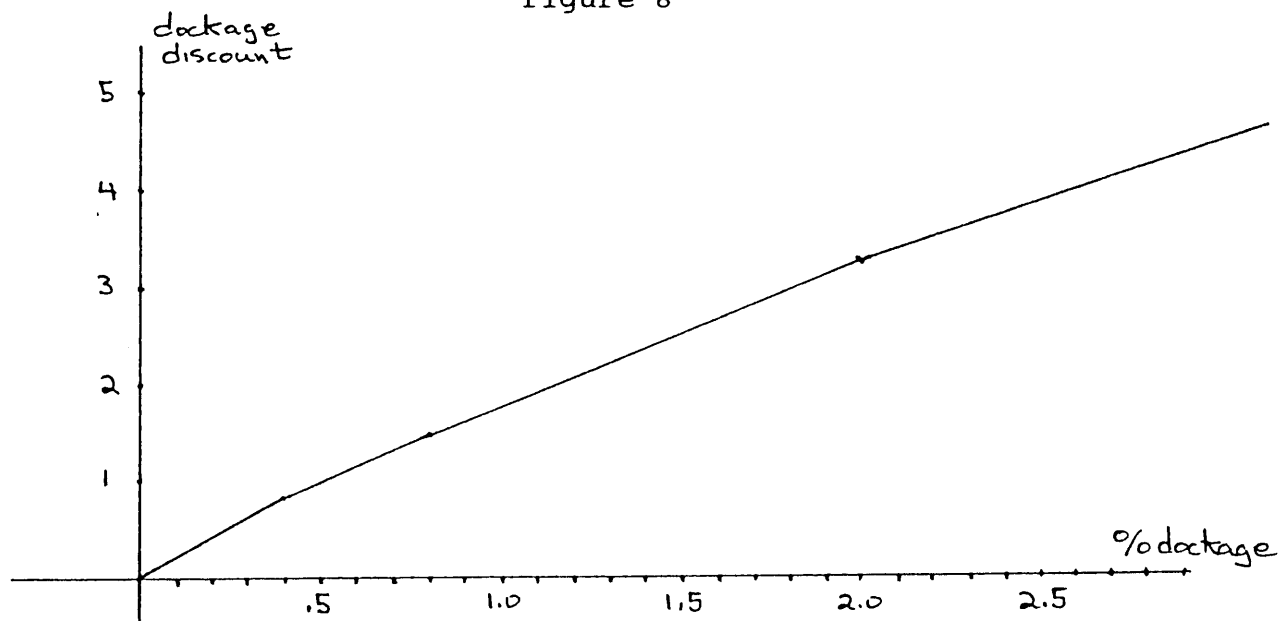
I believe that the wheat industry would be best served by a discount schedule that started at zero dockage and had increments that decline slightly as dockage increases to discourage blending. Since dockage is now routinely measured in .1% increments, the discount schedule should increase with each .1% dockage.

Unfortunately, there does not seem to be a simple way to design such a schedule. However, either of two approaches would work. In either case it may be necessary to measure the dockage penalties in .01¢ increments. First, if there were ranges of

dockage where blending was not detrimental, the schedule could have constant increments within ranges of dockage with the increments declining as dockage increased. For example, a discount schedule such as

.2¢ for each .1% dockage between .1% and .4%  
 .8¢ + .175¢ for each .1% dockage between .5% and .8%  
 1.5¢ + .15¢ for each .1% dockage between .9% and 2.0%  
 3.3¢ + .125¢ for each .1% dockage greater than 2.0%

figure 8



This schedule encourages keeping "clean wheat" with dockage under .4% separate. The ranges and discounts could be adjusted as needed. It is important only that the increase in the discount per .1% dockage not increase as the percentage of dockage increases.

A second possibility would be to use a mathematical formula to generate the discounts. Again, there is no simple formula

whose increments decline as dockage increases. However, there are several more complicated formulas that would have the right properties. For example,

$$2d - .01d^2$$

$$3\sqrt{d} - 1$$

$$10\ln(d + 4) - 14$$

where  $d$  is dockage measured in percent and  $\ln(d + 4)$  is the natural logarithm of  $d + 4$ . See figure 9 and table 1 to get an idea of the schedules that these formulas would produce.<sup>2</sup>

Personally, I like the latter formula since it is closest to constant increments. The industry has not used complicated mathematical formulas or discounts in .01¢ increments. However, since these formulas can easily be programmed into computers and since most settlements are done on computers, using formulas like the ones above may be feasible.

The current dockage discount schedule has some important deficiencies. The schedules proposed above may better meet the wheat industry's need for a fair way to provide cleaner wheat for our customers. Many of the issues discussed here will also arise as the wheat industry addresses other quality needs of our customers -- for example, how to develop incentives to obtain a reliable supply of low protein soft white wheat.

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<sup>2</sup> I developed these formulas by trial and error. I tried to make the discount at 1% dockage close to 2¢ and the increments in that region about .2¢. There are other similar formulas that would also work.

figure 9

# DOCKAGE DISCOUNT SCHEDULES

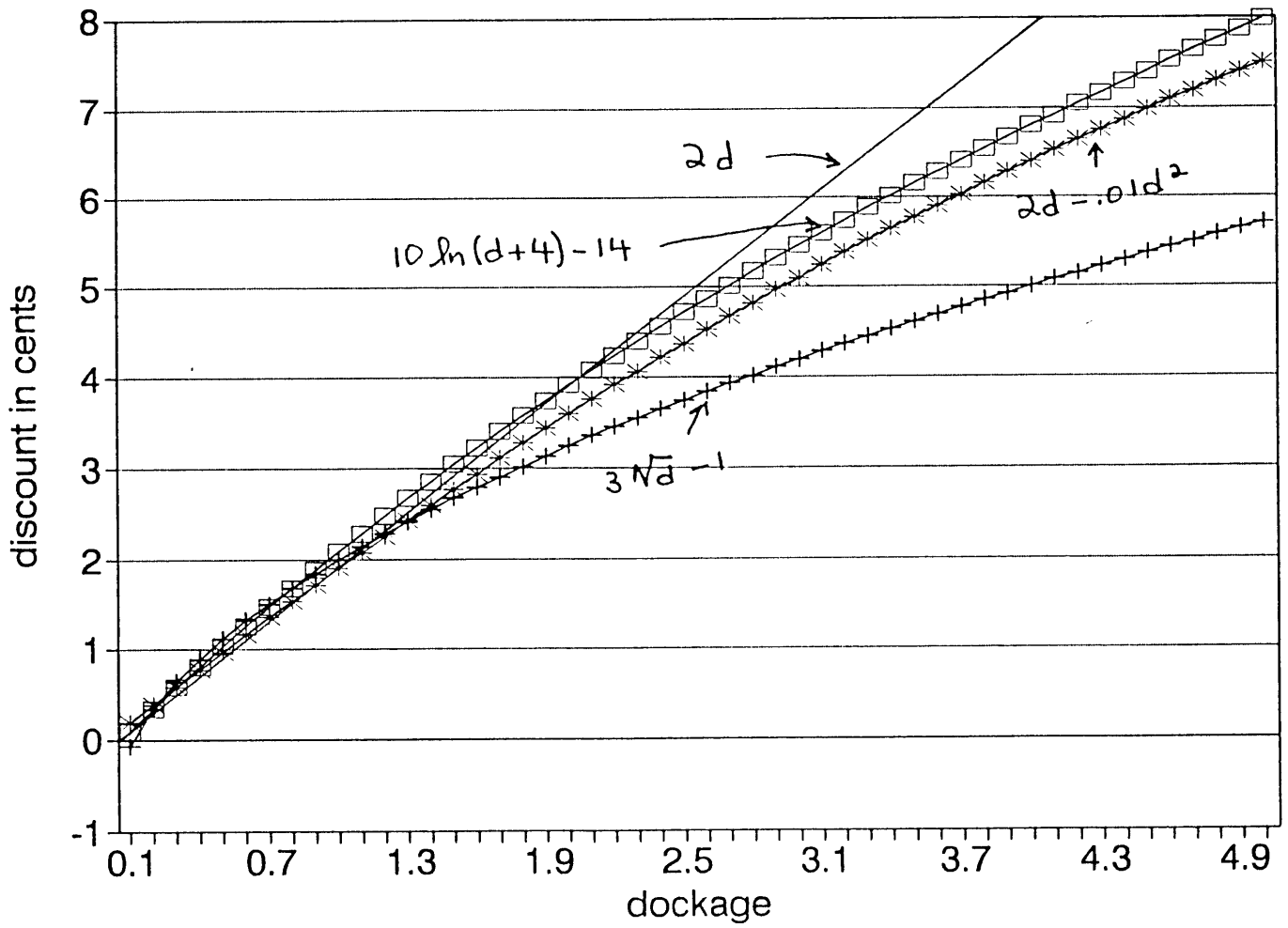




Table 1

POSSIBLE DISCOUNT SCHEDULES  
(cents per bushel)

$d$ Dockage %	$2d$	$3\sqrt{d} - 1$	$2d - .01d^2$	$10 \ln(d+4) - 14$
0.1	0.20	-0.05	0.20	0.11
0.2	0.40	0.34	0.40	0.35
0.3	0.60	0.64	0.59	0.59
0.4	0.80	0.90	0.78	0.82
0.5	1.00	1.12	0.98	1.04
0.6	1.20	1.32	1.16	1.26
0.7	1.40	1.51	1.35	1.48
0.8	1.60	1.68	1.54	1.69
0.9	1.80	1.85	1.72	1.89
1	2.00	2.00	1.90	2.09
1.1	2.20	2.15	2.08	2.29
1.2	2.40	2.29	2.26	2.49
1.3	2.60	2.42	2.43	2.68
1.4	2.80	2.55	2.60	2.86
1.5	3.00	2.67	2.78	3.05
1.6	3.20	2.79	2.94	3.23
1.7	3.40	2.91	3.11	3.40
1.8	3.60	3.02	3.28	3.58
1.9	3.80	3.14	3.44	3.75
2	4.00	3.24	3.60	3.92
2.1	4.20	3.35	3.76	4.08
2.2	4.40	3.45	3.92	4.25
2.3	4.60	3.55	4.07	4.41
2.4	4.80	3.65	4.22	4.56
2.5	5.00	3.74	4.38	4.72
2.6	5.20	3.84	4.52	4.87
2.7	5.40	3.93	4.67	5.02
2.8	5.60	4.02	4.82	5.17
2.9	5.80	4.11	4.96	5.32
3	6.00	4.20	5.10	5.46
3.1	6.20	4.28	5.24	5.60
3.2	6.40	4.37	5.38	5.74
3.3	6.60	4.45	5.51	5.88
3.4	6.80	4.53	5.64	6.01
3.5	7.00	4.61	5.78	6.15
3.6	7.20	4.69	5.90	6.28
3.7	7.40	4.77	6.03	6.41
3.8	7.60	4.85	6.16	6.54
3.9	7.80	4.92	6.28	6.67
4	8.00	5.00	6.40	6.79
4.1	8.20	5.07	6.52	6.92
4.2	8.40	5.15	6.64	7.04
4.3	8.60	5.22	6.75	7.16
4.4	8.80	5.29	6.86	7.28
4.5	9.00	5.36	6.98	7.40
4.6	9.20	5.43	7.08	7.52
4.7	9.40	5.50	7.19	7.63
4.8	9.60	5.57	7.30	7.75
4.9	9.80	5.64	7.40	7.86
5	10.00	5.71	7.50	7.97

## Appendix

The Discount Schedule and Blending

Let

$d_i$  = the fraction of dockage in lot  $i$ .

$Q_i$  = the total number of bushels in lot  $i$ .

$D_i = f(d_i)$  = discount per bushel for dockage fraction  $d_i$ .

Assume a seller has two lots. One lot has dockage  $d_1$  and the other lot has dockage  $d_2$  ( $d_1 < d_2$ ). When is it profitable to blend the two lots and sell the resulting mixture as one lot?

If the two lots are sold separately, the total discount will be

$$(1) \quad f(d_1) \cdot Q_1 + f(d_2) \cdot Q_2$$

If the lots are blended and sold as one lot, the total discount will be

$$(2) \quad f\left(\frac{d_1 Q_1 + d_2 Q_2}{Q_1 + Q_2}\right) \cdot (Q_1 + Q_2)$$

Blending will be profitable if (1) is greater than (2). It is convenient to work with the discount per bushel rather than the total discount. Dividing both (1) and (2) by  $(Q_1 + Q_2)$  and letting

$$(3) \quad \lambda = \frac{Q_1}{Q_1 + Q_2} \quad 0 \leq \lambda \leq 1,$$

blending is profitable if

$$(4) \quad f(\lambda d_1 + (1-\lambda)d_2) < \lambda f(d_1) + (1-\lambda)f(d_2) .$$

It is profitable to keep the lots separate if

$$(5) \quad f(\lambda d_1 + (1-\lambda)d_2) > \lambda f(d_1) + (1-\lambda)f(d_2)$$

and if

$$(6) \quad f(\lambda d_1 + (1-\lambda)d_2) = \lambda f(d_1) + (1-\lambda)f(d_2)$$

the seller is indifferent between blending and segregation.

Proposition 1 - A seller will be indifferent between segregating and blending two lots for all values of  $\lambda$  (i.e. all possible combinations of lot sizes) if and only if the discount function,  $f(\ )$ , is linear between  $d_1$  and  $d_2$ .

Proof:

That indifference implies linearity can be shown as follows:

$$\begin{aligned}\lambda f(d_1) + (1-\lambda)f(d_2) &= f(d_1) + (1-\lambda)[f(d_2) - f(d_1)] \\ &= f(d_1) + \frac{f(d_2) - f(d_1)}{(d_2 - d_1)} \cdot (1-\lambda)(d_2 - d_1) \\ &= f(d_1) + \frac{f(d_2) - f(d_1)}{(d_2 - d_1)} \cdot [\lambda d_1 + (1-\lambda)d_2 - d_1]\end{aligned}$$

so

$$(7) \quad \lambda f(d_1) + (1-\lambda)f(d_2) = \alpha + \beta[\lambda d_1 + (1-\lambda)d_2]$$

where

$$\alpha = \frac{f(d_1)d_2 - f(d_2)d_1}{(d_2 - d_1)}$$

$$\beta = \frac{f(d_2) - f(d_1)}{(d_2 - d_1)}$$

Substituting (7) into (6),

$$(8) \quad f(\lambda d_1 + (1-\lambda)d_2) = \alpha + \beta[\lambda d_1 + (1-\lambda)d_2] \quad .$$

For (8) to hold for all  $0 < \lambda < 1$ ,  $f(\ )$  must be linear.

That linearity of  $f(\ )$  implies indifference can be shown as follows:

Assume

$$(9) \quad f(d) = \alpha + \beta \cdot d \quad \text{where } \alpha \text{ and } \beta \text{ are constants}$$

Substituting (9) into (6),

$$\begin{aligned}\alpha + \beta[\lambda d_1 + (1-\lambda)d_2] &= \lambda(\alpha + \beta d_1) + (1-\lambda)(\alpha + \beta d_2) \\ &= \alpha + \beta[\lambda d_1 + (1-\lambda)d_2] \quad .\end{aligned}$$

So (6) holds for all  $\lambda$ .<sup>1</sup>

Proposition 2 - In order for segregation to be profitable for all  $0 < \lambda < 1$  (i.e. for all combinations of lot sizes),  $f(\ )$  must be strictly concave between  $d_1$  and  $d_2$ .

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<sup>1</sup> This result can also be used to show that for the total discount to be independent of the way the wheat is split into lots, the discount function must be linear.

Proof:

This result follows directly from the definition of a strictly concave function. The function  $f(x)$  is defined as strictly concave if

$$(10) \quad f(\lambda x_1 + (1-\lambda)x_2) > \lambda f(x_1) + (1-\lambda)f(x_2) \quad .$$

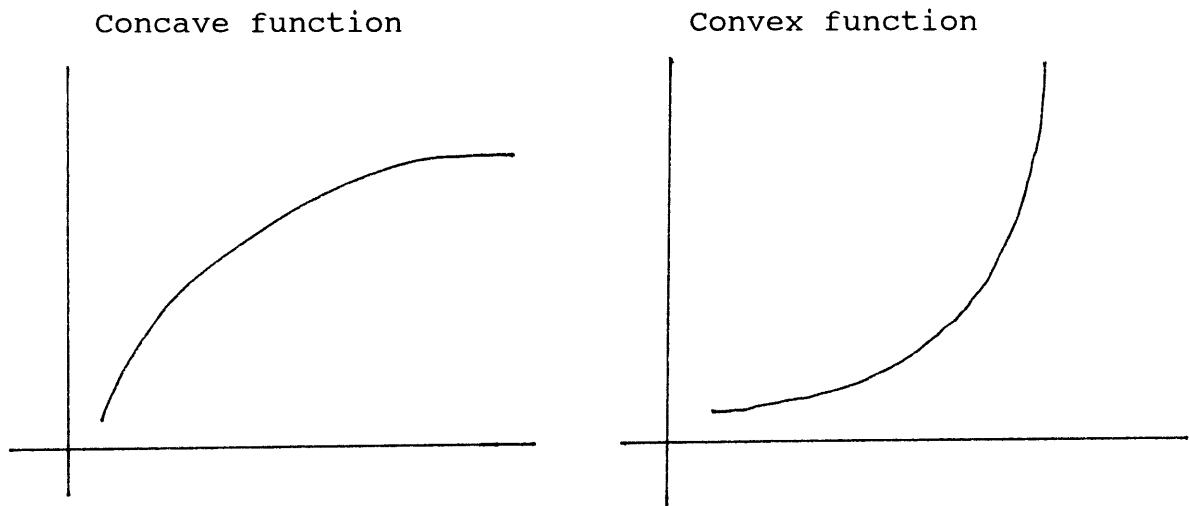
Proposition 3 - In order for blending lots to be profitable for all  $0 < \lambda < 1$ ,  $f(\ )$  must be strictly convex between  $d_1$  and  $d_2$ .

Proof:

This result follows directly from the definition of a strictly convex function. The function  $f(x)$  is defined as strictly convex if

$$(11) \quad f(\lambda x_1 + (1-\lambda)x_2) < \lambda f(x_1) + (1-\lambda)f(x_2) \quad .$$

For a good discussion of concave and convex functions, see Kelvin Lancaster, Mathematical Economics, pages 331-334.



Proposition 4 - Define  $d_A$  as the weighted average of the dockage fractions in the two lots.  $d_A$  is the dockage fraction of the lot that results from blending the two lots,

$$(12) \quad d_A = \lambda d_1 + (1-\lambda)d_2 \quad .$$

Then, if  $f(\ )$  is strictly convex,

$$(13) \quad Q_2 \cdot [f(d_2) - f(d_A)] > Q_1 \cdot [f(d_A) - f(d_1)] \quad .$$

Similarly, if  $f(\ )$  is strictly concave,

$$(14) \quad Q_2 \cdot [f(d_2) - f(d_A)] < Q_1 \cdot [f(d_A) - f(d_1)] \quad .$$

Proof:

Substituting (12) into the definition of a strictly convex function, (11),

$$(15) \quad f(d_A) < \lambda f(d_1) + (1-\lambda)f(d_2) \quad .$$

Substituting  $f(d_A) = \lambda f(d_1) + (1-\lambda)f(d_2)$  into (14) and rearranging,

$$(16) \quad (1-\lambda) \cdot [f(d_2) - f(d_A)] > \lambda \cdot [f(d_A) - f(d_1)] \quad .$$

Substituting (3) into (16) and multiplying both sides by  $[Q_1 + Q_2]$  yields (13).

If  $f(\ )$  is strictly concave, (14) can be derived in a similar fashion with the inequalities reversed.

In words,  $[f(d_2) - f(d_A)]$  is the reduction in the per bushel dockage discount for bushels in the higher dockage lot after blending. Similarly,  $[f(d_A) - f(d_1)]$  is the increase in the per bushel dockage discount for bushels in the lower dockage lot after blending. Multiplying the per bushel reduction by  $Q_2$  gives the total dollars saved by blending the higher dockage lot. Multiplying the per bushel increase by  $Q_1$  gives the total dollars of increased dockage discounts that must be paid because the lower dockage lot is blended. Proposition 4 says that, if the discount function is strictly convex, the savings on the higher dockage lot will exceed the increase in dockage penalties on the lower dockage lot. Hence, blending is profitable. If the discount function is concave, the saving from blending the higher dockage lot are less than the extra dockage penalty on the lower dockage lot, so keeping the lots separate is profitable.

If we consider blending one bushel with dockage  $d_1$  with one bushel with dockage  $d_2$ ,  $d_A$  will be the simple average of the dockages,  $d_A = \frac{1}{2}d_1 + \frac{1}{2}d_2$ . In this case, since  $Q_1 = Q_2 = 1$ , (15) implies that

$$f(d_A) - f(d_1) < f(d_2) - f(d_A)$$

This is one of the results used in the text (see page 7).